

1.)

a) What are the expected value, standard deviation and shape of the sampling distribution of the p -bar

Expected Value / mean = 0.25

$$\begin{aligned}\text{Standard deviation} &= \sqrt{p \frac{q}{n}} \\ &= \sqrt{0.25 \times \frac{0.75}{7.5}} \\ &= 0.05\end{aligned}$$

b) Probability that the sample proportion will be greater than 0.34

$$P(Z > 1.8) = 0.0359$$

$$P = \frac{(0.34 - 0.25)}{0.05} = \frac{0.09}{0.05} = 1.8$$

c) What is the probability that the sample proportion will be between 0.196 and 0.354

$$\begin{aligned}P &= \left(\frac{0.196 - 0.25}{0.05} \right) < Z < \left(\frac{0.354 - 0.25}{0.05} \right) \\ &= 0.8412\end{aligned}$$

2) A bank has kept records of the checking balances of its customers & determined that the average daily balance of its customers is \$ 300 with a standard deviation of \$ 48. A random sample of 144 checking accounts is selected.

a) What is the probability that the sample mean will be more than \$ 306.60?

$$P(306.6 \leq \bar{x})$$

$$\Rightarrow P\left(\frac{306.6 - 300}{48} \leq \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - 300}{4}\right)$$

$$(1.65 \leq z) = 0.5 - P(0 \leq z \leq 1.65) = 0.5 - 0.4505 = 0.0495$$

b) What is the probability that the sample will be less than \$ 308?

$$P(\bar{x} \leq 308)$$

$$P = \frac{308 - 300}{48}$$

$$P(z \leq 2) = 0.5 + P(0 \leq z \leq 2) = 0.5 + 0.4772 = 0.9772$$

c) What is the probability that the sample mean will be between \$ 302 and \$ 308

$$P(302 \leq \bar{x} \leq 308)$$

$$P\left[\frac{302 - 300}{48} \leq \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - 300}{4} \leq \frac{308 - 300}{48}\right]$$

$$P(0.5 \leq z \leq 2) = P(0 \leq z \leq 2) - P(0 \leq z \leq 0.5) = 0.4772 - 0.1915 = 0.2857$$

d) What is the probability that the sample mean will be at least \$ 296

$$P(296 \leq \bar{x})$$

$$= P\left[\frac{296 - 300}{48} \leq \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - 300}{4}\right]$$

$$P(-1 \leq z) = 0.5 + P(-1 \leq z \leq 0)$$

$$= 0.5 + 0.3413$$

$$= 0.8413$$

... the checking balances

d) What is the probability that the sample proportion will be less than 0.25

~~0.5000~~ $p = 0.5000$

e) What is the probability that the sample proportion will be less than 0.10

$$z = \frac{0.10 - 0.25}{0.05} = -3 \text{ by empirical rule}$$

$$P(z < -3) = \frac{(100 - 99.7)}{2} = 0.15\%$$

z-table gives 0.0044

2	87	Pass		
3	92	Pass		
4	35	Fail		
5	79	Pass		
6	100	Pass		
7	48	Fail		
8	74	Pass		
9	79	Pass		
10	91	Pass		
	mean	75		
	standard dev	20.48305533		
	Variance	419.5555556		
	Pass	8		
	Fail	2		
	Total	10		
	Proportion=Pass	0.8		

Q4.

a) $\mu = 110,000$, $\sigma = 36,000$
 $n = 81$
 $\bar{X} = 120,000$

$$H_0: \mu \geq 110,000$$

$$H_1: \mu < 110,000$$

b) Test Statistic.

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{120,000 - 110,000}{36,000/\sqrt{81}} = 2.5$$

c) Critical Value

$$@ 95\% \text{ C.L.} = 1.96$$

Decision rule: Reject H_0 if test statistic $>$ critical value.

Q4 a)

$$x = 80$$

$$n = 400$$

$$\hat{p} = \frac{80}{400} = 0.2$$

$$C.I = \hat{p} \pm Z \cdot \sqrt{\frac{(1-\hat{p})\hat{p}}{n}}$$

$$= 0.2 \pm 1.96 \left(\sqrt{\frac{(1-0.2)0.2}{400}} \right)$$

$$= 0.2 \pm 0.0392$$

$$(0.1608, 0.2392)$$

b) M.E = Critical value $\times$ Standard error of statistic

$$0.04 = 1.96 \times \sqrt{\frac{0.2(1-0.2)}{n}}$$

$$\left(\frac{0.04}{1.96} \right)^2 = \left(\sqrt{\frac{0.16}{n}} \right)^2$$

$$\left(\frac{0.04}{1.96} \right)^2 = \frac{0.16}{n}$$

$$n = \frac{0.16}{\left(\frac{0.04}{1.96} \right)^2}$$

$$= 584.16 \approx 584$$